Interest Rates and Currency Prices in a Two-Country World

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Contribution

Integrates domestic and international monetary theory with financial economics to provide a complete theory of equilibrium goods and securities pricing for a two-good two-country economy.
Important Abstractions

1. **No Home Bias**: all traders in the world hold identical portfolios (crucial to the results)
2. **No Production**: with money it would introduce a wedge between private and social returns to L and K, because workers would want to trade effort today for consumption tomorrow
3. **No Precautionary Saving**: unit-velocity of money, know exactly how much and at what price to buy, currency exactly to match
4. **No “business cycle”**: monetary instability has no effect on real movements.
Outline

• Section 2: A Barter Model

• Section 3: Monetary Model

• Section 4: Two-Currency Flexible Exchange Rate Model

• Section 5: Fixed Exchange Rate Model
Barter Model

Assumptions:

1. Trader in both countries have identical preferences over infinite stream of consumption goods
2. Non-storable endowments follow a Markov process
3. Agents are risk averse
4. Trade in both goods in spot and in advance, before the realization of the shocks
5. Rational expectations: know the probability distribution of future prices
Barter Model

Equilibrium:

1. Equilibrium prices are Arrow-Debreu, determined by the appropriate marginal rates of substitution
2. Agents perfectly pool market risks, owning as a result half of claims to “home” endowment and “half” to foreign
3. Each trader begins and ends a period with the identical portfolio of equity claims
4. Agents in both countries begin with equal wealth and maintain this situation over time
5. All private portfolios together claim all real output
Barter Model

Agents’ optimization problem:

\[ v(\theta, s) = \max_{x, y, \theta_x, \theta_y} \{ U(x, y) + \beta \int v(\theta', s') f(s', s) ds' \}, \tag{2.4} \]

s.t.

\[ x + p_y(s)y + q_x(s)\theta_x + q_y(s)\theta_y \leq \theta. \]

\[ \theta' = \theta_x[\xi' + q_x(s')] + \theta_y[p_y(s')\eta' + q_y(s')]. \]
Barter Model

Equilibrium Prices:

Spot prices are determined by marginal rates of substitution in a perfectly pooled equilibrium:

$$p_y(s) = U_y(\frac{1}{2} \xi, \frac{1}{2} \eta)/U_x(\frac{1}{2} \xi, \frac{1}{2} \eta) = U_y(s)/U_x(s).$$  \hspace{1cm} (2.10)

Model yields equity pricing formulas in an efficient market. (2.11) and (2.12) are analogous to non-constant growth dividend valuation models of equities.

$$q_x(s) = \beta [U_x(s)]^{-1} \int U_x(s') [\xi' + q_x(s')] f(s', s) \, ds'.$$  \hspace{1cm} (2.11)

$$q_y(s) = \beta [U_y(s)]^{-1} \int U_y(s') [p_y(s') \eta' - q_y(s')] f(s', s) \, ds'.$$  \hspace{1cm} (2.12)
Monetary Model

Use of money motivated by CIA constraint
Assume that securities earn positive nominal rate of return

Timing of Trading:
1. \( S = (\xi, \eta) \) and monetary shocks realize
2. Traders from both countries engage in perfectly competitive securities trading using their securities and currency holdings
3. Trade and goods and currencies (i.e. use currency to finance goods purchases)
Monetary Model

Agents’ optimization problem:

Note: M not a choice variable because positive nominal interest rates collapse “goods demand” and “currency demand” into a single problem, since agents will hold non-interest bearing assets in the exact amount needed to cover current period goods purchases.

\[ v(s, w, \theta) = \max_{x, y, \theta_x, \theta_y} \{ U(x, y) + \beta \int v(s', w', \theta') \, dF \, dH \} \]  \hspace{1cm} (3.9)

s.t

\[ x + p_y(s)y + q_x(s, w)\theta_x + q_z(s, w)\theta_y \leq \theta. \]  \hspace{1cm} (3.7)

\[ \theta' = \frac{\xi' + p y(s)\eta'}{(1 + w')(\xi + p y(s)\eta)} \left[ \xi \theta_x + p_y(s)\eta\theta_y \right] + q_x(s', w')\theta_x + q_z(s', w')\theta_y + \frac{1}{1 + w'}(\xi' + p_y(s)\eta'). \]
Equilibrium Result 1:

Money is NOT neutral because monetary-fiscal policies affect relative prices. Here monetary transfers have “differential effect” on equity prices, because these depend on joint distribution of money and real shocks (i.e. get arbitrary correlations between money transfers and real shocks).

\[
U_x(s)q_x(s, w) = \beta \int U_x(s') \left[ q_x(s', w') + \frac{\xi' + p'y(s')\eta'}{1 + w'} \cdot \frac{\xi}{\xi + p_y(s)\eta} \right] dF \, dH,
\]

(3.15)

\[
U_x(s)q_y(s, w) = \beta \int U_x(s') \left[ q_y(s', w') + \frac{\xi' + p'y(s')\eta'}{1 + w'} \cdot \frac{p_y(s)\eta}{\xi + p_y(s)\eta} \right] dF \, dH.
\]

(3.16)
Monetary Model

Equilibrium Result 2:

When money is unstable it is no longer possible for all private portfolios to claim all real output: must pay inflation tax. Because with money shocks dividends are discounted by monetary transfers.

\[ q(s) = \beta [U_x(s)]^{-1} \int U_x(s')[q(s') + \xi' + p_y(s')\eta'] \, dF. \]  \hspace{1cm} (3.20)

vs.

\[ q(s, w) = \beta [U_x(s)]^{-1} \int U_x(s')[q(s', w') + \frac{\xi' + p_y(s')\eta'}{1 + w'}] \, dF \, dH. \]  \hspace{1cm} (3.21)
Monetary Model

Equilibrium Result 3:

Under a stable monetary policy a single dollar-denominated bond is the equivalent of a fully diversified equity claim to “world output” one period hence.

\[ \beta [U_x(s)]^{-1} \int U_x(s') [\xi' + p_y(s') \eta'] dF. \]

This expression is identical to the ‘dividend’ term in the equity price formula (3.21), when \( w' \equiv 0 \).
Monetary Model With Exchange Rate

- Have two national currencies with two different rates of growth:

\[ M_{t+1} = (1 + w_{0,t+1})M_t, \quad (4.1) \]
\[ N_{t+1} = (1 + w_{1,t+1})N_t, \quad (4.2) \]

this allows agents to pool monetary risks as well as endowment risks. Thus get currency securities.

- Exchange rate determined according to PPP:

\[ e(s, M, N) = p_x(s, M)p_y(s)[p_y(s, N)]^{-1} = \frac{M \eta}{N \zeta} p_y(s). \quad (4.5) \]
Monetary Model With Exchange Rate

- Optimization problem is identical to section 3, except get two additional choice variables thus two additional security prices

New budget constraint:

\[ x + p_y(s)y + r_x(s, w)\psi_x + r_y(s, w)\psi_y + q_x(s, w)\theta_x + q_y(s, w)\theta_y \leq \theta. \] \hspace{1cm} (4.9)

Two new security prices:

\[ U_x(s)r_x(s, w) = \beta \int U_x(s') \left[ r_x(s', w') + \frac{w'_0}{1 + w'_0} \xi' \right] dF dK, \] \hspace{1cm} (4.15)

\[ U_x(s)r_y(s, w) = \beta \int U_x(s') \left[ r_y(s', w') + \frac{w'_1}{1 + w'_0} p_y(s')\eta' \right] dF dK. \] \hspace{1cm} (4.16)
Fixed Exchange Rate Model

- Exchange rate maintained at a constant level through C.B. intervention in currency market
- C.B. holds reserves of both currencies, trading in spot currency markets so as to maintain the exchange rate at constant value
- Rule out speculative attacks by assumption; i.e. behavior of central authority in combination with monetary policy and real shocks in both countries is consistent with fixed exchange rate under rational expectations

Total reserves:

$$D - R + \bar{e}S$$

the equilibrium exchange rate becomes

$$\bar{e} = \frac{M - R \eta}{N - S \xi} p_x(s).$$
Fixed Exchange Rate Model

- Viability of fixed exchange rate requires $R>0$, $S>0$ for all possible states $(s, M, N)$. Using (5.1) and (5.2) this implies:

\[
D > N\bar{e} - M\frac{\eta}{\xi} p_y(s), \quad \text{and} \quad (5.3)
\]

\[
D > M - N\bar{e}\left/\frac{\eta}{\xi} p_y(s) \right.. \quad (5.4)
\]

Thus $D$ must be greater than the value of all dollars outstanding and all pounds outstanding. (5.3) and (5.4) cannot be maintained indefinitely if $M$ and $N$ drift over time, hence fixed exchange rate regime requires coordination between monetary policies of both countries.
Fixed Exchange Rate Model

Equilibrium Prices:

To derive equilibrium spot and equity prices recognize that:

\[ M_t - R_t + e(N_t - S_t), \] or ‘world money’ plays the role of \( M_t \) in section 3.

Assuming finance constraint binds, will get identical results as in section 3. Moreover, if money is perfectly stable then private portfolios can claim all real output and resource allocation is Pareto-Optimal; i.e. get full list of Arrow-Debreu contingent claim securities.