Stochastic Herding by Institutional Investment Managers

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Motivation

- Strategic behavior and choice correlations important to outcomes in financial markets (Morris and Shin (1999))
- Herding eventually leads to self-organized criticality and market crashes (Johansen, Sornette, Ledoit (1999))

⇒ Micro-level imitation leads to macro-level coordination
⇒ How to identify and quantify such behavior?
Approach

- Importance of interactions between agents (complex systems)
  - strategic complementarity
  - know individual optimization problems
    ⇒ total outcome is probabilistic
    ⇒ chain reaction through information revelation about market liquidity

- Analysis by distribution
  - accumulation of risk captured by distribution parameters
  - economic interpretations

Data

13F Filings with SEC

- Thompson Financial Spectrum database
- Institutional investment managers with over $100 million under management
- 16 quarters 2003:Q1 - 2008:Q1
- S&P 500 stocks
- Types: Banks and Trusts, Insurance Comp., Invest. Comp., Invest. Adv., All Other (Pension, Endowment Funds)
The Sellout Phase

\[ \alpha(j, k) = \frac{\text{# investors type 'j' selling > 80% of stock 'k' in quarter 't'}}{\text{investors in the group}} \]
Normality Test of $a(j, k)$ over the entire sample, 2003:Q1 - 2008:Q1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(j, k)$</td>
<td>38,353</td>
<td>22.745</td>
<td>79.264</td>
<td>6.368</td>
<td>54.868</td>
</tr>
</tbody>
</table>

- $\alpha(j, k)$ has large kurtosis, highly non-normal
- inverse of slope provides good approximation of the mean – exponential distribution
  - LS regression in S&P 500 stocks $\hat{\beta} = -31.443$ (std. 0.055, $R^2$ 0.988)
Why Exponential?

- Normality is a good hypothesis: central limit theorem should hold if investors act independently.
- Not likely that exogenous aggregate shocks follow an exponential distribution.
- Exponential tail occurs in a domino effect.
- – an indication of herding?
Model

- $N$ informed traders indexed by $i = 1, 2, \ldots, N$ endowed with one unit of risky asset
- $(g - r)p$ gains for staying invested in the bubbles
- $\beta p$ loss if the bubble bursts.
- trader $i$ can either sell ($a_i = 1$) or remain in the same position ($a_i = 0$)
- observed: aggregate number of informed selling traders $a \equiv \sum_{i=1}^{N} a_i$, (corresponding fraction $\alpha = a/N$)
- unobserved: liquidity provided by noise traders $\theta$
- bubble bursts if $\alpha > \theta$
- private signal: $\chi_i = \theta + \epsilon_i$
- prior belief on $\theta$ and $\epsilon_i$ jointly follow a bivariate normal distribution with mean $(\theta_0, 0)$ and variance $(\sigma_\theta^2, \sigma_\epsilon^2)$
Optimal strategy

\[(g - r) \Pr(\theta \geq \alpha \mid x_i, a, a_i = 0) - \beta \Pr(\theta < \alpha \mid x_i, a, a_i = 0) = 0\]  
(1)

or, equivalently, sell out if:

\[\frac{g - r}{\beta} < \frac{\Pr(x_i, a, a_i = 0, \theta < \alpha)}{\Pr(x_i, a, a_i = 0, \theta \geq \alpha)}\]  
(2)

Threshold rule: trader \(i\) sells if \(x_i \leq \bar{x}(a)\) and holds otherwise. \(\bar{x}(a)\) implicitly determined by (2).
Then the optimal strategy is to sell if:

\[ x_i < \bar{x} \]

where \( \bar{x} \) is constant.

Then \( a \) follows a binomial distribution with probability \( Pr(\bar{x} > x_i) \) and population \( N \). This asymptotes to normal when \( N \) is large.
Endogenous feedback

By MLRP there exists a threshold private signal $\bar{x}$ given $a$ that divides optimal action space into two regions. Case shown for $a' > a$. 
Under the threshold rule, the joint probability has three parts:

1. Private information $x_i$

$$\frac{\Pr(x_i, \theta < \alpha)}{\Pr(x_i, \theta \geq \alpha)}$$

2. Information revealed by holding actions of $N - 1 - a$ informed traders

$$\left( \frac{\Pr(x_j > \bar{x}(a) \mid \theta < \alpha)}{\Pr(x_j > \bar{x}(a) \mid \theta \geq \alpha)} \right)^{N-1-a}$$

3. Information revealed by selling actions of $a$ informed traders

$$\left( \frac{\Pr(x_j \leq \bar{x}(a) \mid \theta < \alpha)}{\Pr(x_j \leq \bar{x}(a) \mid \theta \geq \alpha)} \right)^a$$
Equilibrium selection

- Multiple equilibria may exist for a realization of \((x_i)\)
- We assume that the market maker selects the smallest \(\alpha\)
- The selection alters the information revealed by sellers as:

\[
\prod_{k=0}^{a-1} \frac{\Pr(x_j \leq \bar{x}(k) \mid \theta < \alpha)}{\Pr(x_j \leq \bar{x}(k) \mid \theta \geq \alpha)}
\]
Equilibrium

- \((x_i, \alpha)\) pair such that each trader has no incentive to deviate from the threshold rule given that all the other traders obey the rule.

- Equilibrium thresholds show strategic complementarity:

\[
\begin{align*}
N &= 160, \ g = 0.1 \\
r &= 0.04, \ \beta = 0.82 \\
\theta &= \text{N}(0.5, 0.3) \\
\epsilon_i &= \text{N}(0, 1)
\end{align*}
\]
\( \mu(a) = \frac{H'}{H} \frac{d\bar{x}(a)}{da} (N - a) \) (3)

\[ H(\bar{x}) = \int_{\bar{x}} e^{-\frac{(x_j - \theta_1)^2}{2\sigma_e^2}} dx_j / \sqrt{2\pi\sigma_e} \]

- \( \mu(a) \) is Poisson mean of number of traders induced to sell by trader \( a \)
- \( \frac{H'}{H} \) : hazard rate for the traders who have remained holding the asset to sell upon observing \( a \)
- \( \frac{d\bar{x}(a)}{da} \) is of order \( 1/N \)
Let $\mu_1 \equiv \mu(0)$ denote the mean number of traders that would sell in response to shocks $x_i$ even when $a = 0$.

Suppose that $\mu(a)$ is constant at $\mu$ for $a > 0$.

Then $a$ follows a distribution with exponential tail:

$$\Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a!$$

$$\sim (\mu e^{1-\mu})^a a^{-1.5}$$

Slope of the exponential tail is determined by $\mu$. 

$$\Pr(a)$$
Probability Density: Borel-Tanner Distribution

\[ \Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a! \]

\[ \mu_1 = 2.060 \]
\[ \mu = 0.547 \]
Numerical Simulations

- $\mu(a)$ is not assumed to be constant

- $a$ is distributed similarly to an exponential distribution for $0 < a < 50$. There is no incident of $a > 50$ except for the 691 “explosive” incidents in which case basically all the traders decide to sell.
Numerical Simulations

- Left blowups of the histogram for $0 < a < 160$. Right semi-log plot.
Evidence from institutional transactions

- *Left* histogram of empirical $a(j, k)_t$. *Right* semi-log probability plot of empirical $a(j, k)_t$
- Fitted distribution with ML parameter estimates

\[
\Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a!
\]

\[
\Pr(a) = (\mu e^{1-\mu})^a a^{-1.5}
\]
Evidence from institutional transactions

Table: Distribution parameter estimates for $a(j, k)$ for the 2005:Q2 - 2006:Q1 subsample.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Distribution of $a(j, k)$</th>
<th>Benchmark Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Borel-Tanner</td>
<td></td>
<td>Trunc. Normal Gamma Exponential</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>2.058</td>
<td>mean -97.461</td>
<td>$\alpha$ 1.103 $\beta$ 4.781</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(7.152) (0.021)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.570</td>
<td>$\sigma$ 20.000</td>
<td>$\beta$ 4.335</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.665) (0.103)</td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>11148.789</td>
<td>10040.186</td>
<td>10925.596 10938.238</td>
</tr>
<tr>
<td>Vuong’s statistic</td>
<td>$H_1$</td>
<td>30.393</td>
<td>21.785 28.140</td>
</tr>
<tr>
<td>Obs.</td>
<td>4,265</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The probability density for the hypothesized distribution is

$$Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a!$$
Evidence from institutional transactions

Parameter interpretation:

- $\mu_1 = 2.058$: 2 managers within each group would have sold the stock even if no one else was selling.
- $\mu = 0.570$: 57% chance that a fund manager would have chosen to follow the actions of another fund manager.
- $\text{Fit } e^{-\beta a(j,k)} \equiv \mu e^{(1-\mu)a(j,k)}$ with an exponential distribution: $\beta = 0.19$
- $\hat{\beta}$ is 0.209
Exponential Decay and the Rise of $\mu$ Over Time

Figure: Semilog probability plot of $a(j, k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.
Exponential Decay and the Rise of $\mu$ Over Time

Figure: Herding – quarterly estimates of distribution parameter $\mu$, which measures the probability of a “chain reaction” in response to a random liquidation by an investment manager. Initial independent liquidations occur with Poisson arrival rate of $\mu_1$. The probability density of the aggregate action is then given by $\Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a!$. 
Illiquidity

The key feature: stock’s illiquidity, $1/\theta$, propagates herding. Specifically, negative (positive) relationship between the intensity of the branching process, $\mu$, and the realized liquidity (illiquidity) of a security. We defined:

\[ H(\bar{x}) = \int_{\bar{x}} e^{-\frac{(x_j - \theta_1)^2}{2\sigma_e^2}} dx_j / \sqrt{2\pi\sigma_e}. \]  

Then, we obtain:

\[ H'(\bar{x}) = -e^{-\frac{(\bar{x} - \theta_1)^2}{2\sigma_e^2}} / \sqrt{2\pi\sigma_e} < 0, \]  

\[ \frac{d}{d\theta_1} H'(\bar{x}) = H'(\bar{x})(\bar{x} - \theta_1)/\sigma_e^2. \]

Thus, $H'(\bar{x})$ is strictly increasing for any $\bar{x} < \theta_1$. From Equation (17), we have:

\[ \mu(a) = (H(\bar{x}(a - 1)) - H(\bar{x}(a)))(N - a). \]

Therefore, we obtain $d\mu(a)/d\theta_1 < 0$ for any $a$ such that $\bar{x}(a) < \theta_1$. 
Figure: *Left:* $\mu$ as a function of $a$ in Equation 17 for different realizations of liquidity $\theta$. *Right:* simulated histograms of $a$ for high and low realizations of $\theta$. 

\[ e_1 = 0.7 \quad e_1 = 0.3 \]
Illiquidity

$1/\theta_1$: Amihud (2002) proxy for a stock's realized illiquidity

$$\Pr(\alpha(j, k)) = \mu_1 e^{-\left(\mu_{j,k} \alpha(j,k) + \mu_1\right) \left(\mu_{j,k} \alpha(j,k) + \mu_1\right)^{\alpha(j,k)-1}/\alpha(j,k)!}$$

(10)

and

$$\mu_{j,k} = \gamma_0 + \gamma_1 1/\theta_1(j) + \epsilon_{j,k}.$$  

(11)

We pick trivial hyper-parameters for the priors:

$$\gamma_i \sim N(0, 1), \quad i = 0, 1$$

(12)

$$\mu_1 \sim N(0, 1)$$

(13)

$$\tau_\epsilon \sim \Gamma(\alpha_\epsilon, \beta_\epsilon)$$

(14)
## Illiquidity

### Table: Illiquidity and the Empirical Proxy for Herding, $\mu$

<table>
<thead>
<tr>
<th>Dependent parameter: $\mu_{j,k}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>node</td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>$1/\theta_1(j)$</td>
</tr>
<tr>
<td>$\mu_1$</td>
</tr>
<tr>
<td>$\tau_\epsilon$</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

Notes: Results of Bayesian MCMC estimation of hierarchical model in Equation (10). $\Pr(a) = \mu_1 e^{-(\mu a + \mu_1)}(\mu a + \mu_1)^{a-1}/a!$ with restriction $\mu = \gamma_0 + \gamma_1 1/\theta_1 + \epsilon$. Higher $\mu$ indicates higher intensity of the branching process generating the Borel Tanner distribution. Results based on 89,000 samples after discarding the first 11,000 iterations as “burn-in”. Confidence bounds computed under the normality assumption for the simulated parameter values. 2003:Q1 through 2006:Q1 time sample.
The *fraction* of dumping institutions follows an exponential distribution before the regime change.

Micro-founded model of herding in exit decisions generates exponential tail.

Simulations show reasonable correspondence to institutional ownership data.

The *number* of dumping institutions follows a distribution consistent with the model.

In line with “market-timing” data generating mechanism, the distribution parameter measuring the degree of herding is increasing in a stock’s illiquidity as well as rises sharply prior the sell-out phase.